Projecting delay and compression of mortality

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(joint work with J. de Beer & F. Janssen)

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Motivation

- the importance of mortality delay and compression,
- literature on mortality projection using age-at-death distribution is rare,
- compression is not steady over all ages,
- modal age increases in parallel with life expectancy at birth.

Simulation of mortality delay

Simulation of mortality compression

Objectives

-Present a model that decomposes delay and compression.

-Projecting mortality delay and compression.

-Distinguish mortality compression in young, adult, and advanced ages.

-Comparison with Lee-Carter model.

-Case studies: Japan, France & USA (both genders).

-Data: from 1960-2014 and ages from 0-100.

The (Co)mpression (De)lay model (CoDe)

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- Non-linear model.
- Parameters constraints:
  $0 < A_t < 1, B > 0, 0 < \alpha_t < 1, 0 < g < 1, \beta_{1,t} > 0, \beta_2 > 0, \beta_{3,t} > 0 \,\&\, h > 0.$
- We present the CoDe 2.1 version.
- Continuous version of the CoDe model (de Beer & Janssen, 2016).
- Describes the full age pattern of mortality and assesses mortality delay and compression.
- Applied to unsmoothed data from HMD.

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- Death probabilities at very old ages ($x \to +\infty$) are equal to g.
- Trivial extrapolation to ages beyond age sample.

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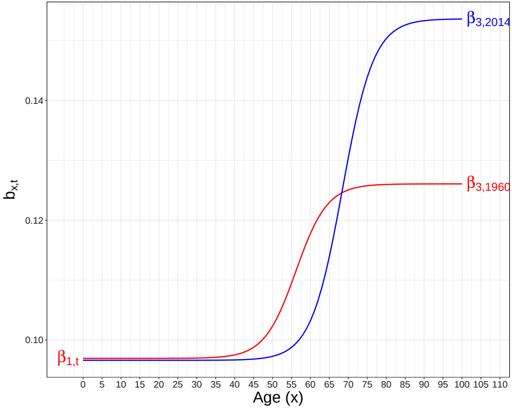
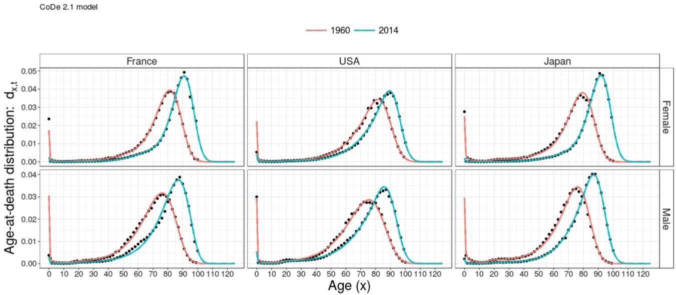


Fit of CoDe 2.1 model: d_x



- For the fitting of CoDe 2.1 we used the Differential Evolution (DE) algorithm (`RcppDE` package in `R`).

Projections

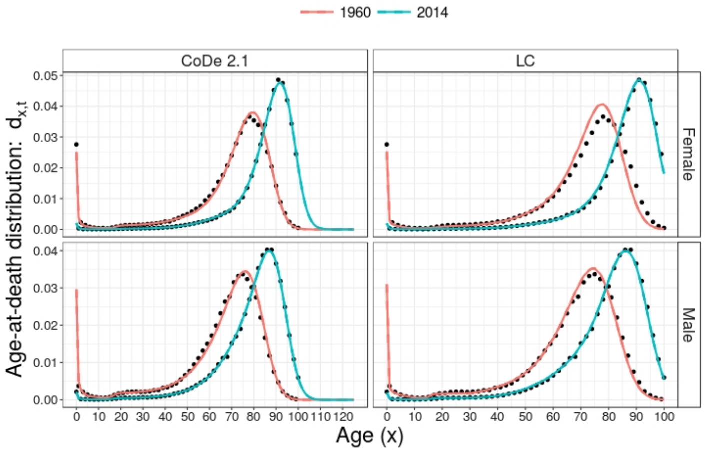
For the CoDe 2.1 model:

- For the projections of time dependent parameters we used ARIMA models.
- Dependency is established through dependent errors, modeled by a multivariave normal distribution.
- To ensure that the corresponding projections will satisfy the constraints we transformed the parameters (e.g logit A_t).
- Out of sample projections 40 years (i.e. 2014-2054).
- Backtesting: three periods
  i. Short (5 years), ii. Middle (10 years), iii. Long (15 years).

For the LC model:

- We used SVD to fit the model and RW to project the time dependent parameter.

In-sample comparison: Japan d_x.



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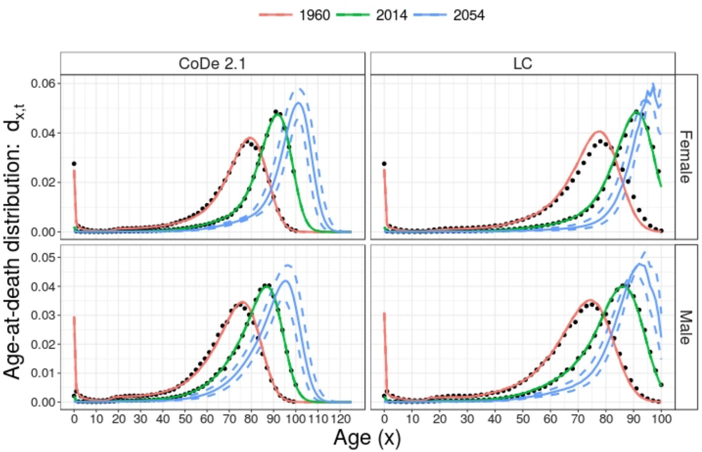
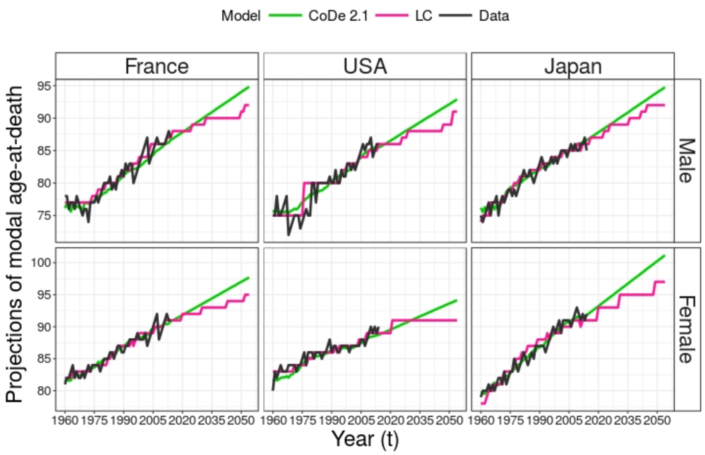


Out-of-sample comparison: Modal age (M_t).



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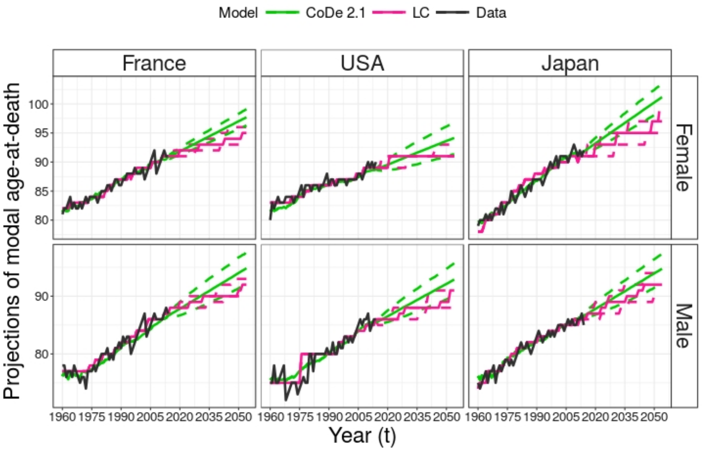
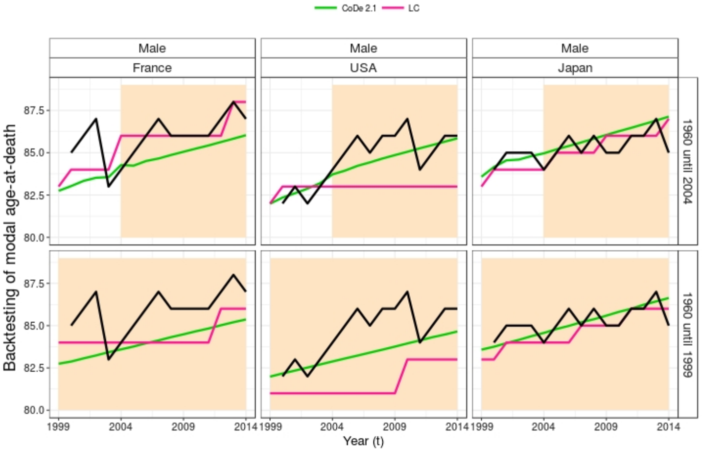


Backtest comparison: Males M_t.



Conclusion

When there is both compression and delay, the Lee-Carter model **projects a slowdown of delay**, whereas the CoDe 2.1 model projects a continuation of delay.

Future work

- comparison with other mortality models,
- a multi-population version of the CoDe model,
- apply CoDe model to cohort data,
- include smoking, alcohol and obesity epidemics,
- R package,
- ...

Thank you for your attention!

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www.futuremortality.com